Math 5375–001, Fall 2016:
Constructing Whole Number and Operations

5–8 PM Tuesdays, Room 311 PKH, UTA

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Office Hours: before and after class and by appt.

Prerequisites: Graduate standing and consent of instructor
Text materials: DMI's *Building a System of Tens* and *Making Meaning for Operations* casebooks (henceforth BST and MMO); *Children's Mathematics: Cognitively Guided Instruction* by Carpenter et al. (CGI), see bibliography. Addl. materials will be provided in class or on the course web page.
Course home page: http://mathed.uta.edu/kribs/5375.html
Last day for withdrawal: November 2

LEARNING OUTCOMES: The successful student will be able to:

- identify and use (in teaching) common conceptual models of the four arithmetic operations;
- identify, analyze and explain (justifying in simple terms) both traditional and invented computational algorithms for the four arithmetic operations;
- identify and use appropriate representations of whole numbers and place value in teaching situations, including conceptual, contextual, concrete, pictorial, and symbolic models;
- design and implement research-based lessons to teach whole number concepts, including the development, selection and sequencing of problems, as well as appropriate assessment;
- anticipate and identify common approaches & errors in student learning of number and operation concepts.

FORMAT: This course will study number and operation concepts in several ways: through work on challenging mathematical problems to develop our own mathematical abilities (and communicating the results, to develop our expository abilities); discussing what research has discovered about the learning of number and operation concepts at the K–8 levels; and examining specific instances of K–8 students' computational work, through both case studies and our own classroom practice.

Before class each week you will read articles and/or case studies from K–8 mathematics, and make notes on them in preparation for class discussions. You will also often work on mathematics problems outside of class, to facilitate their discussion in class (especially important if you often do not complete them in class before we discuss them in large group). We typically begin class by working on new mathematics problems and discussing their solutions, in both small and large groups. We follow this up by discussing the assigned readings, as well as other topics related to problem solving. We typically end class with time for reflection on how the topics we have discussed apply to our own classrooms. During class discussions we often refer back to work we have done earlier in the course, so please bring your notes and papers from previous sessions to class.

POLICIES:

- Students who are not classroom teachers will need to make arrangements (I can help) to interact with K–8 students for many of the assignments (those starred * on the calendar).
- Students are expected to be on time, prepared and ready to work every week. This class meets every Tuesday from 5 PM to 8 PM from August 30 to December 13. Each student is allowed the equivalent of one week's absence (3 total hours) for whatever reason without penalty. All subsequent absences (including arriving significantly late) will result in the reduction of the final course grade by one-half letter grade (5%) for each absence (even if you tell me ahead of time). *Students who miss an entire class must talk with the instructor prior to the next class.* See DMI handout for policy on making up work from any missed class.

• With the exception of diagrams, equations, and student work samples, written assignments should be typed and use correct grammar and punctuation. No cover pages necessary.

• Each student is allowed one late submission during the semester. The paper must be submitted before the beginning of the class period following that in which it was due. Papers not submitted by the end of class time on the due date are considered late. Submission of a late paper constitutes the student's agreement that this is the one allowed late assignment.

• Each student is allowed one electronic (PDF) submission during the semester. Electronic submissions must be complete including any ancillary materials such as student work necessary for grading. (If the electronic submission is made late, then it is both the only late paper allowed and the only electronic submission allowed.) This does not include drafts sent for consultation.

• Each student is allowed to submit one revised paper for a regrade, under the following terms: The revised paper and the graded original must be turned in together at Session 15. The new grade replaces the original. Students should consult with the instructor before submitting a revision.

• As a sign of respect for your peers and our common work, please keep all cellular phones, computers, and other electronic devices turned off during class. In emergencies cell phones may be set to vibrate only, and brief calls taken in the hallway outside.

• This course follows University policies on topics such as drops, withdrawals, academic integrity, accommodating disabilities, etc. Please see the attached supplement for further details.

GRADES: Your grade for the course will be determined by five elements, each of which has equal weight: (1) journal entries and participation, (2) a written student interview, (3) a short case study, (4) a paper detailing your own mathematical work, and (5) a lesson involving a problem you select to develop numerical concepts in your students. All of these are detailed in the next section.

Assignments

1. Journal

On days in which there is not a major assignment due, you will write a short (about one page) reflection in response to a prompt given below. Many of the prompts are also given in the DMI handouts distributed in class. Some involve "action research" reports in which you will write about your own students' mathematical work. You will often use and discuss your responses in class, within your small groups and in large group. These reflections are to be turned in at the end of class; I will respond to them in writing and return them at our next class meeting. Grading will be limited to verifying that responses are appropriate in topic and scope (length).

Your journal entries will serve to document your preparation for class each day (and your growth over time); this component of your grade will be based half on your journal (entries should be complete each day before class) and half on your participation in class discussions (I expect participation in large-group discussion at least ten of the fifteen times we will meet).

J1 *Children's definitions for addition and subtraction.* Ask several students to define or explain what addition and subtraction are (use age-appropriate vocabulary, but be careful not to use terms like putting together and taking away, which already do most of the defining). Report their responses verbatim, and then compare them with the conceptual models for these two operations identified in the readings. Do their definitions cover all the conceptual models?

J2 *Mini-interview.* Interview a single student about a single question related to the meanings and structure of one of the four arithmetic operations or place value (at an appropriate level for the student). Give the student's response and analyze the understanding the student shows. The intent of this assignment is as a dry run for the student interview assignment, so see that portion of the syllabus for the general format, but keep in mind that this mini-interview should cover only a single question and thus be much smaller in scope.

J3 *Division and remainders.* (i) Write six different simple story problems for the division $32 \div 5$, each of which has a different one of the following expressions as answer: (a) 6 remainder 2, (b) $6\ \frac{2}{5}$, (c) 6.4, (d) 6 or 7, (e) 6, (f) 7. (ii) Write a simple story problem or question that students could use to think about why dividing by zero is meaningless.

J4 *Mini-case study.* Pose, to a group of K–8 students, a question similar to those in the cases we have read or seen, or from one of our class discussions. (Make sure it is appropriate to the students' grade level.) Write about your question, what you expected, and what actually happened. Did anything surprise you? Please describe specific examples of what your students say and do. Examining the work of a few students in detail may be more helpful than trying to incorporate the responses of every student. Think of this as a dry run for the case study.

J5 *Defining place value.* First, ask several students to define or explain what we mean by place value. Report their answers, and then give your own definition. What differences do you see?

J6 *Choosing strategies I.* Explain in detail a student's work on a computational problem where the student used a different strategy than the traditional algorithm (preferably from your class, otherwise from one of the DMI case studies—be sure to explain the strategy in any case). Then consider the skills required and not required by this computational approach, and speculate why the student chose that strategy instead of the more traditional approach.

J7 *Choosing strategies II.* Learners develop and choose computational strategies based on features of the problem being solved, as well as their own levels of understanding of various relevant concepts. Investigate whether or not students make these choices *consciously*, by revisiting some students' work with them, and asking them why they chose the approaches they used.

J8 *Article review.* Choose an article from one of the three NCTM practitioner journals which deals with student learning in the number and operation strand, and write a review of it including its implications for teaching. Give a full bibliographic citation. (Back issues of these journals are available at the UTA libraries; each journal also has one sample issue available for free review on www.nctm.org.)

J9 *One problem paper: Analyzing multiplication strategies.* Each of the following three computations uses a nontraditional multiplication algorithm to reach a correct answer. For each computation, give a rigorous (justified) analysis including answers to the following questions:

(1) Is it mathematically sound?

(2) If so, how far can it be extended?

(3) Based upon the skills required and not required (relative to the traditional algorithm), what motivated the approach?

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(a)    2 4   (b)   7 2 5    (c)   1 2 9 0
     x 6 4         x   8        x   4 0 3
     2 5 6         6 6 0        3 6 2 7 0
 + 1 2 8       + 5 1 4 0    + 4 8 3 6 0
   1 5 3 6       5 8 0 0      5 1 9 8 7 0
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J10 *Synthesis.* Looking back on the readings and discussions of the course to date, write a paragraph on each of the following topics:

- the key issues in developing conceptual understanding and computational fluency in operating on whole numbers;
- the problem, discussion or issue that most changed your own mathematical understanding (not your students');
- the problem, discussion or issue that most changed the way you teach (or will teach) number and operation topics;
- the role of invented & traditional algorithms and drill in developing computational fluency.

2. Student interview

In order to develop (or strengthen) the habit of attending to student thinking in detail, you will conduct an interview with a student from your class to assess her/his understanding of a specific mathematical topic. You may choose the student and topic, but the interview should involve a topic from number and operation. Begin by obtaining all necessary permissions to conduct and record (audio or video) the interview; explain to all interested parties (including the student!) that you need the student's help for a class in which you are studying how students learn, and this interview will not affect her/his grades; it will just help you understand how (s)he thinks. (Recording the interview will keep you from needing to make detailed notes during it.)

Before the interview, get a copy of recent written work by the student showing her/his ability to reason and problem-solve (the work need not be error-free, but there should be enough progress made to discuss the problem). Make sure the student is familiar with the paper, and begin the interview by asking him/her to explain the work, including what difficulties s/he encountered.

Continue the interview by asking further questions about the mathematical topic involved (see the handout on interviewing tips on the course web site). You will need to use both pre-prepared questions and ad hoc follow-up questions to develop a coherent line of questioning. Remember that *in order to determine the limits of a student's knowledge, you must continue until you reach a question which the student either cannot answer or answers incorrectly for reasons other than a simple careless error.* You should be able to do this without making the student feel badly.

After the interview, use your recording to make a more detailed analysis of the student's thinking, with regard to both problem-solving abilities and knowledge of the particular mathematical topic. Begin with a brief introduction to provide context. Give an overall narration of the interview (e.g., say what specific tasks or problems you asked the student to work on). Use specific details or quotes to support your analysis. Conclude your write-up with an explicit summary of what the student knows, what the student does not know, and what the student is ready (or needs) to work on next (see interview tips handout for more).

3. Case study

During the course we will read and discuss in class several case studies, all describing events in other teachers' classrooms. For this assignment, you are to write a short (roughly 3–5 pages) case study describing a mathematical discussion involving one or more students, similar to these cases. A case is neither a complete transcript of a lesson nor as prefabricated as an interview, although it is very helpful to include direct quotes and dialogue from students.

You must base your case on a conversation for which you were present, and preferably in which you were involved, but it could come out of a lesson you observed, or a conversation among two or more students. You may choose to narrow in on one or two students, or on one small group, or you may describe a whole-class conversation. The most important thing is that the episode illustrate some aspect of children's mathematical thinking, as well as your reflection on larger teaching issues it raises for you. It must also center on a mathematical topic involving number and operation.

In writing your case study, begin by describing briefly the class's larger context (including grade level) and the mathematical topic; then report the relevant parts of the conversation in as much detail as you can manage. Include your thoughts as you work with the students. Finish by summarizing your evaluation of the students' thinking and identify what issues and questions the case raises for you. *It is important that your reflection address teaching issues beyond the one topic and set of students involved*, in order to document your ability as a reflective practitioner to make connections that inform your teaching practice more broadly.

We will discuss the writing of cases in more detail before they are due, but you are encouraged to begin sooner if you have a good conversation fresh in your mind. I will be glad to help you.

4. 2-problem paper

In order to understand the concepts underlying number and operation (including teaching it), you must gain experience in explaining its applications. As a summative evaluation of the mathematical portion of this course, you will submit a paper detailing your mathematical work on a *college-level* problem from this course which you solved completely, and a problem from K–8 mathematics which you believe is related. You *must* check the course web page or meet with me individually to approve and verify the problem you wish to write up.

For the college-level problem, give a thorough explanation of the original problem (paraphrased), its context, the strategies you used to approach it, what the solution is (and why! that's the tricky part), and what the solution means in context. Distinguish carefully between conjectures and rigorous arguments. Feel free to use drawings, graphs, diagrams, tables, etc. if necessary. *Cite outside sources clearly.*

Also select a single problem (*not* a lesson) from K–8 mathematics (possibly multi-part, and preferably from your own classroom) which you believe entails number and operation concepts similar to those involved in the college-level problem, and explain in detail what those concepts are, clarifying in a paragraph what common ideas the two problems share. Include the prompt in your paper. *Limit your analysis to properties of the problems themselves; do not focus on student work (unlike in the other major papers).*

I encourage you to show me a draft of your paper before final submission.

5. Lesson paper

In this course we will study the teaching and learning of ideas related to number and operation in K–8 mathematics. As a summative evaluation of the pedagogical aspects of this course, you will develop or select an exemplary lesson which fosters learning these concepts, teach and document the lesson, and give a short (10-minute) presentation to the class on it. The lesson draft checkpoint includes items 1–3 below. The final lesson paper you submit must include *all* of the following components:

1. Select or develop *a single, rich problem* intended for use with the students you teach, which involves a major idea from this course. You may use or adapt a problem from class materials, but be sure it is appropriate for the target audience. Present the prompt in a paragraph. (Say where you got it from, and, if you have used it before, in what capacity, and what you learned from it.) The best lessons tend either to integrate multiple strands of mathematics to illustrate connections, or to address significant conceptual issues within a single strand as a summative activity following multiple experiences in developing and exploring a concept. Specify prerequisite knowledge.
2. Write a paragraph explaining what concepts from this course are entailed in this problem. (You may use deconstruction if it helps you identify them, but write in paragraph form.)
3. Add to the above written descriptions a short sketch (in paragraph form) of how you plan to use the problem in a lesson, and meet with your instructor to discuss your progress.
   (These 3 paragraphs form the lesson draft checkpoint. They will also form part of your final paper.)
4. Write a formal lesson plan that uses the problem/activity as a significant problem-solving opportunity with your students. Include closure activities, and important discussion points.
5. Teach the lesson to your students (see me if this is problematic). Then write a one-page reflection on how the lesson went, including what strategies students used to approach the problem, what ideas were raised in its discussion, and to what extent your students' understanding of the underlying whole number concepts—or ability to apply them—changed as a result of the lesson. Be specific.
6. Make a one-page handout (you may use front and back if necessary, but it *must* fit on one sheet) summarizing your lesson for the class. Include the problem, grade level, mathematical topics addressed, and anything your colleagues would need to know in order to use the lesson, including (briefly) any difficulties the students tended to encounter. The handout should *not* be the same as your lesson plan (select details!), and must be turned in with the main paper.
7. Give a brief (10-minute) presentation to the class on this lesson, using the handout, at our last class meeting.

I encourage you to discuss this project with me as often as you like, throughout the semester. A preliminary draft of the modified problem and lesson idea (not [necessarily] yet taught) is due at Session 10 (see step 3 above). Final documentation is due at Session 15 (so that I can return it to you), including a handout, with the presentations to be given at Session 16.

Bibliography

Rebecca Ambrose, Jae-Meen Baek, and Thomas Carpenter. (2003). Children's invention of multidigit multiplication and division algorithms, in A. J. Baroody and A. Dowker (Eds.), *The development of arithmetic concepts and skills: constructing adaptive expertise*. Lawrence Erlbaum Associates, Mahwah, N.J. pp. 305–336.

James K. Bidwell. (1991). Susan's personal algorithm. *Arithmetic Teacher* 39(3): 1.

Carpenter, T.P., Fennema, E., Franke, M.L., Levi, L., Empson, S.B. (1999). *Children's Mathematics: Cognitively Guided Instruction*. Heinemann/NCTM, Portsmouth, NH.

Carpenter, T.P., Franke, M.L., Jacobs, V., & Fennema, E. (1998). A longitudinal study of invention and understanding in children's multidigit addition and subtraction. *Journal for Research in Mathematics Education* 29: 3–20.

Guershon Harel and Merlyn Behr. (1991). Ed's strategy for solving division problems. *Arithmetic Teacher* 39(3): 38–40.

Deborah Schifter, Virginia Bastable, and Susan Jo Russell. (2010). *Building a System of Tens (Number and Operation, Part 1) Casebook*. 2nd edition. Boston, MA: Pearson.

Deborah Schifter, Virginia Bastable, and Susan Jo Russell. (1999). *Making Meaning for Operations (Number and Operation, Part 2) Casebook*. Parsippany, NJ: Dale Seymour/Pearson.

Calendar

A tentative schedule with topics is given below (subject to updating).

Sess.	Date	Topic	Readings/Cases Due	Assignments Due
1	8/30	Early concepts of operations (MMO1)	MMO1, CGI1	—
2	9/06	Models for addition and subtraction (MMO2)	MMO2, CGI2,3	J1* defn. add/sub
3	9/13	Models for multiplication (MMO3)	MMO3, CGI4	J2* mini-interview
4	9/20	Models for division	CGI5	J3 divisions
5	9/27	Numeration systems	BST Case 8, [num sys]	Interview*
6	10/04	Place value and Mayan numeration (BST2)	BST2+old Case 13	J4* mini-case
7	10/11	Bases in place value	—	J5* defn. place value
8	10/19	Algorithms for multidigit add. and sub. I (BST1)	BST1	Case study*
9	10/25	Algorithms for multidigit add. and sub. II	BST optional	J6 choosing I
10	11/01	Grouping and regrouping in add. and sub.	CGI6	Lesson draft, J7*
11	11/08	Algorithms for mult. of multidigit numbers	Ambrose et al.	J8 article review
12	11/15	Partial factors in multiplication (BST4)	BST4, CGI7	J9 mult. algs.
13	11/22	Algorithms for div. of multidigit numbers (BST5)	BST5	J10 synthesis
14	11/29	Division by multidigit numbers	CGI8, Bidwell, Harel	2-problem paper
15	12/06	Invented and traditional algorithms in learning (BST3)	BST8:1–4,MMO8:1–4, BST3,CGI Appendix	Lesson paper*
16	12/13	Final presentations	—	Give presentations

{BST/MMO/CGI}n means Chapter n of the given book.
See bibliography for further details of readings (nonelectronic readings are available at the UTA Libraries).